Ion Landau Damping on Drift Tearing Modes

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The equations governing the ion Landau damping (ILD) layers for a drift tearing mode are derived and solved to provide a matching to ideal MHD solutions at large x and to the drift tearing solution emerging from the ion kinetic region, $k\rho_i \sim 1$, at small x, the distance from the rational surface. The ILD layers lie on either side of the mode rational surface at locations defined by $k_y x V_{Ti}/L_s = \omega_{*e}(1 + 0.73\eta_e)$ and have been ignored in many previous analyses of linear drift tearing stability. The effect of the ILD layer on the drift tearing mode is to introduce an additional stabilizing contribution, requiring even larger values of the stability index, Δ' for instability, than predicted in refs.[1, 2]. The magnitude and scaling of the new stabilizing effect in slab geometry is discussed.

I Introduction

Tearing modes are driven by current gradients and pressure gradients in the plasma, whose destabilising effect is characterized by a quantity Δ'. The associated tearing of the magnetic field occurs at the resonant surface, r_0, where $m = nq(r_0)$, m and n being the poloidal and toroidal mode numbers of the helical instability. In the vicinity of $r = r_0$ complex kinetic effects can occur. To account for these it is convenient to treat a narrow 'inner' region in which these effects are included but the equations are simplified by virtue of the localised nature of this layer. Stability is then determined by matching solutions of these inner equations to outer ones, valid across the regions away from the layer (and satisfying appropriate boundary conditions at the plasma boundary and magnetic axis) where a simple ideal MHD description of the plasma suffices. The solution of the inner

equations provides a quantity $\Delta(\omega)$, where ω is the mode frequency, while the solution of the outer equations yields the, aforementioned, quantity Δ': the matching condition leads to a dispersion relation.

For a simple resistive MHD model, (or even a cold ion model containing electron diamagnetic effects) of the inner region, the corresponding solutions match satisfactorily to the outer ones, with the perturbed parallel electric field, $E_\parallel$ vanishing as one reaches the outer ideal MHD region. However, for a kinetic treatment of ion physics, when finite ion Larmor radius, FLR, and parallel thermal motion are included, this matching becomes more complex. Even though one can match the functional dependence of the longitudinal component of the perturbed vector potential, $A_\parallel$ from the inner region to its form in the ideal MHD region, where $A_\parallel \sim 1 + (r - r_0)\Delta'$, the inner region expression for $E_\parallel$ fails to vanish as it should. In order to accomplish this (i.e. decay of $E_\parallel$) within the matching range, one must consider an intermediate region where ion sound and ion Landau damping become significant at the characteristic frequency of a drift tearing mode. As a consequence the overall matching condition is modified with an impact on the stability of the drift tearing mode. The investigation of this effect in a cylindrical plasma model is the purpose of this paper.

II Plasma Model

We start from the fourth order coupled set of equations[1], in slab geometry, comprising the vorticity equation:

$$k_\parallel J_\parallel = \frac{e^2 n\phi}{T_i}\left[(1-\Gamma_0(b))(\omega-\omega_{*i}) - \omega_{*i}\eta_i b(\Gamma_0-\Gamma_1)\right], \tag{1}$$

and Ampère's equation,

$$\frac{d^2 A_\parallel}{dx^2} = -J_\parallel, \tag{2}$$

where ϕ and $A_\parallel$ are the perturbed electrostatic potential and longitudinal component of the perturbed vector potential, $k_\parallel = k_y x/L_s$ is the longitudinal wavenumber, with x the distance from the mode rational surface and L_s shear length: in toroidal geometry $L_s = Rq/s$, with R the major radius, $s = rq'/q$ and q the safety factor. In eqs.1,2, $J_\parallel = J_{\parallel,i} + J_{\parallel,e}$ is the

perturbed current density, e, the proton charge, n, the equilibrium density, ω the mode frequency, $\omega_{*j} = k_y T_j/(e_j B L_n)$ the diamagnetic frequency of each species, $1/L_n = dLog(n)/dx$, $\eta_j = dLog(T_j)/dLog(n)$; $\Gamma_n(b) = I_n(b)e^{-b}$, $b = k_\perp^2 T_i/(m_i\omega_{ci}^2)$, with I_n the modified Bessel functions; $k_\perp^2 = k^2 + k_y^2$, denotes the component of the wave number perpendicular to the magnetic field, with k_y the component of $k_\perp$ within the magnetic surface, and k the component perpendicular to the surface. Finally, ω_{ci} denotes the ion cyclotron frequency.

Expressions for the perturbed current densities, $J_{\|,e}$ and $J_{\|,i}$ are derived from the respective kinetic equations for electrons and ions (in the $k\rho_i \ll 1$ limit):

$$J_{\|,e} = -i\frac{e^2 n E_\|}{k_\|^2 T_e}(\omega - \omega_{*e}), \tag{3}$$

$$\begin{aligned} J_{\|,i} = -i\frac{e^2 n E_\|}{k_\|^2 T_i}\,&[(\omega - \omega_{*i})(1 + \zeta_i Z(\zeta_i)) \\ &-\omega_{*i}\eta_i(\zeta_i^2(1 + \zeta_i Z(\zeta_i)) - \frac{1}{2}\zeta_i Z(\zeta_i))\Big] \end{aligned} \tag{4}$$

where $\zeta_i = \omega L_s/k_y x V_{T_i}$, $V_{T_i} = \sqrt{2T_i/m_i}$, Z is the plasma dispersion function. Making use of the long wavelength approximation for the ions in eq.1, with $b \sim \rho_i^2 d^2/dx^2 \ll 1$, eliminating $J_{\|,i,e}$ and introducing a dimensionless variable $X = 1/\zeta_i$, we obtain the following fourth order system describing the potentials ϕ, $A_\|$, which we now denote by A.:

$$X\frac{d^2A}{dX^2} = \beta_i(1 - \frac{\omega_{*i}}{\omega})\frac{d^2\hat{\phi}}{dX^2}, \tag{5}$$

$$\begin{aligned} X\frac{d^2A}{dX^2} &= \frac{\beta_e}{2}\left(\frac{L_s}{L_n}\frac{\omega}{\omega_{*e}}\right)^2\left(\hat{\phi} - \frac{A}{X}\right)\left[1 - \frac{\omega_{*e}}{\omega} + \tau\left(1 - \frac{\omega_{*i}}{\omega}\right)\left(1 + \frac{Z}{X}\right)\right. \\ &\left. -\tau\frac{\omega_{*i}}{\omega}\frac{\eta_i}{X^2}\left(1 + \frac{Z}{X} - \frac{X\ Z}{2}\right)\right] \end{aligned} \tag{6}$$

where $\hat{\phi} = \phi/V_{Ti}$, $Z = Z(1/X)$, $\tau = T_e/T_i$ and β_j are the ratios of plasma to magnetic pressure for each species.

III Solving the fourth order system through the Ion Landau region.

To solve the fourth order system of eqs.5 and 6 we first exploit the small parameter $\beta_i \ll 1$ in eq.5 to obtain;

$$A_\| \sim 1 + \frac{\Delta'}{2}x \ = \ 1 + \frac{\Delta'}{2}\frac{\omega L_s}{k_y V_{T_i}}X \tag{7}$$

which matches correctly to the ideal MHD outer solution as $X \to \infty$. Then, inserting this result in the RHS of eq.6, we use eq.5 to generate the following second order inhomogeneous ODE for $\hat{\phi}$:

$$\frac{d^2\phi}{dX^2} = Q(X)\left(\phi - \frac{1}{X} - \frac{\Delta'}{2}\frac{\omega L_s}{k_y V_{T_i}}\right), \tag{8}$$

where,

$$Q(X) = \frac{L_s^2}{2L_n^2}\frac{(\hat{\omega}\tau)^2}{\hat{\omega}\tau + 1 + \eta_i}\left[\hat{\omega} - 1 + (\hat{\omega}\tau + 1)(1 + \frac{1}{X}Z) + \eta_i\left(\frac{1}{X^2} + \frac{1}{X^3}Z - \frac{Z}{2X}\right)\right], \tag{9}$$

$$\hat{\omega} = \frac{\omega}{\omega_{*e}}.$$

and we have dropped the caret on ϕ for simplicity.

The objective is now to solve eq.8 throughout the ion Landau region where $X \sim 1$ in order to determine the asymptotic form of ϕ at small values of X, corresponding to $\rho_i \ll x \ll \rho_i(L_s/L_n)$. In this region $\phi(X)$ will match to the drift tearing solution emerging from the ion-kinetic region, where $x \sim \rho_i$. This also takes the form $\phi \sim (a + 1/x)$ and in most earlier analyses of drift tearing modes, e.g.[2], $a = \Delta'/2$ has been assumed; however this requires justification. For this purpose we assume that the relevant mode frequency is known and is characteristic of the drift tearing mode in the semi-collisional electron limit[2], i.e. $\omega = \omega_{*e}(1 + 0.73\eta_e)$.

Noting that $Q(X) \sim O(\tau^2 L_s^2/L_n^2) \gg 1$ throughout the region, we solve eq.8 in the WKB approximation so that the full solution is constructed from the two WKB solutions of the homogeneous eqn.:

$$\phi_{\pm}(X) = \frac{1}{Q^{1/4}}\left[e^{+\int_0^X \sqrt{Q(y)}dy},\ e^{-\int_0^X \sqrt{Q(y)}dy}\right]. \tag{10}$$

The relevant solution, satisfying the matching condition to ideal MHD for $X \gg 1$, takes the form:

$$\phi(X) = \frac{1}{2}\left\{\phi_+(X)\int_X^\infty r(y)\phi_-(y)dy + \phi_-(X)\right\}, \tag{11}$$

$$r(y) = Q(y)\left[\Delta'\frac{\omega L_s}{2k_y V_{T_i}} + \frac{1}{X}\right] \tag{12}$$

In eq.11 the factor 1/2 comes from the Wronskian, $W(\phi_+, \phi_-) \equiv -2$ and c and X_0 are arbitrary constants. In practice it will be convenient to choose a value of $X_0 \ll 1$ when evaluating the asymptotic form of $\phi(X)$ in the $X \ll 1$ region, in the next section.

IV Asymptotic form of ϕ in the small X limit.

In order to match to the drift tearing mode solution in the region $x \gg \rho_i$ we require the functional form of $\phi(X)$ in the region $1/\sqrt{Q} \ll X \ll 1$. In this limit, the ion Z-functions can be approximated by the "ion-sound" expansion with the result:

$$Q(X) \sim \frac{L_s^2}{2L_n^2}\frac{(\hat{\omega}\tau)^2}{\hat{\omega}\tau+1+\eta_i}\{\hat{\omega}-1-\frac{X^2}{2}(\hat{\omega}\tau+1+\eta_i)\}, \tag{13}$$

$$= Q_0\left[1-\lambda X^2\right], \tag{14}$$

$$Q_0 = \frac{1}{2}\left(\frac{\hat{\omega}\tau L_s}{L_n}\right)^2\frac{\hat{\omega}-1}{\hat{\omega}\tau+1+\eta_i}, \qquad \lambda = \frac{1}{2}\frac{\hat{\omega}\tau+1+\eta_i}{\hat{\omega}-1} \tag{15}$$

Now, making use of these results within eq.11, we find that $\phi_-(X) \sim e^{-\sqrt{Q_0}X}$ is exponentially small so that the second term in eq.11 (containing the constant c and the integration end point X_0) can be neglected in the evaluation of ϕ. The integral in the first term of eq.11 is evaluated by successive partial integrations, leading to the result:

$$\phi(X) \sim \left[\frac{\Delta'}{2}\frac{\omega L_s}{k_y V_{Ti}} - \frac{\lambda}{\sqrt{Q_0}}\right] + \frac{1}{X}, \tag{16}$$

to first order in the small parameter, $1/\sqrt{Q_0} \sim L_n/L_s$

We translate this result back into an "effective" value of Δ' to be used for the matching of a drift tearing solution, to an outer (in x) solution. When this matching is performed within the ion Landau region, one finds this effective Δ' is given by:

$$\Delta'_{eff}\rho_i = \Delta'\rho_i - \left(\frac{L_n}{L_s}\right)^2\left(\frac{\hat{\omega}\tau+1+\eta_i}{\hat{\omega}-1}\right)^{3/2}\frac{2}{(\hat{\omega}\tau)^2}, \tag{17}$$

where, as noted earlier, $\hat{\omega} = 1 + 0.73\eta_e$ for a drift tearing mode.

V Discussion and Conclusions

The result obtained in eq.17 implies that the ion sound/ion Landau region introduces a stabilising effect for drift tearing modes. This is in addition to the diamagnetic stabilisation found by Drake et al.[3] and the finite ion orbit stabilisation found by Cowley et al.[1] and

in [2]. The resulting stability threshold for drift tearing can now be expressed in the form:

$$\Delta'\rho_i > \Delta_{FLR} + \Delta_{DIA} + \Delta_{ILD}, \tag{18}$$

$$\Delta_{FLR} = \sqrt{\pi}\hat{\beta}\frac{(\hat{\omega}-1)^2(\hat{\omega}+1)(\hat{\omega}+1-\eta_i/2)}{4\hat{\omega}^2}\left[log(Y)-\frac{\pi}{4}\right], \tag{19}$$

$$\Delta_{DIA} = \pi\hat{\beta}(\hat{\omega}-1)|\bar{I}|, \tag{20}$$

$$\Delta_{ILD} = \left(\frac{L_n}{L_s}\right)^2\left(\frac{\hat{\omega}+1+\eta_i}{\hat{\omega}-1}\right)^{3/2}\frac{2}{(\hat{\omega})^2} \tag{21}$$

where $Y \sim (\rho_i/\delta) \gg 1$ with δ the width of the electron current layer, $\bar{I}(\hat{\omega}, \eta_i)$ is an integral defined in eq.24 of [2] (see also Fig.4 of [2]), $\hat{\beta} = (\beta_e/2)(L_s^2/L_n^2)$ and for simplicity we have set $\tau = 1$. The expressions in eqs.19 and 20 apply when the parameter, $\hat{\beta} < 1$. At high values of $\hat{\beta}$, complete screening of the resonant surface occurs resulting in elimination of the drift tearing mode. We conclude that the additional stabilization represented by Δ_{ILD} can only be of significance at very small values of β_e or at small values of η_e.

Similar "ion sound" modifications of the effective Δ' for drift tearing modes have previously been derived by Bussac et al.[4] and by Fitzpatrick[5] using fluid treatments. In ref.[4], semi-collisional drift tearing modes were considered in the small Larmor radius limit, $\rho_i \ll \delta$. Slab geometry was assumed and temperature gradients were (initially) neglected. The resulting expression for the stability threshold takes the form:

$$\Delta'\rho_i \sim \beta_e\left[\frac{\omega_*}{\nu_e}\frac{L_n}{L_s}\right]^{1/2}, \tag{22}$$

and it was argued that the stabilizing effect becomes much weaker in the presence of an electron temperature gradient, $\eta_e \neq 0$. This same trend is apparent in eq.21 above, where the limit $\eta_e = 0$ appears to be singular. This limit would require a calculation of $\delta\hat{\omega}$, a small correction to the drift tearing frequency, $\hat{\omega}_0 = 1$, as was carried out in ref.[4] in the $\rho_i \ll \delta$ limit. We do not pursue this special case, where $\eta_e = 0$, but note that this explains the differing results in eqs.21 and 22. In [5] toroidal effects were included and analytic solutions generated in a transition region, between the electron current channel and the MHD region, by introducing, as a simple model, a Heaviside function switch-on of the perturbed ion current. Using this device a result of the form:

$$\Delta'_{eff} = \alpha\Delta' + \delta, \tag{23}$$

was obtained, where the multiplier, α is positive, greater than unity, but reduces to unity in the slab limit. The offset term, δ, found by Fitzpatrick is positive (destabilising), but

vanishes in the slab limit.

The foregoing analysis differs from both these treatments in being a kinetic treatment including finite ion orbit effects and in which ion Landau damping can occur. However, the modification to Δ' generated by the ion transition region is not complex; i.e. the imaginary part of the ion Z function never features in the above analysis. The apparent cause of this can be identified by investigating the variation of the longitudinal electric field, $E_{\|}$ through the ion transition region. It appears that $E_{\|}$, which must tend to zero as the outer ideal MHD region is approached, actually becomes small as the ion sound region is traversed by the drift tearing eigenfunction, i.e. before the ion Landau resonances are encountered. This indicates that a fluid treatment of the ion transition region should suffice.

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